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Macroscopic Superpositions and Entanglement of Mesoscopic Squeezed Vacuum States in Dissipative Cavity QED System

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Abstract A scheme has been proposed for generating the macroscopic superpositions and the entanglement between the mesoscopic squeezed vacuum states by considering the two-photon interaction of N two-level atoms in a dissipative cavity with high quality factor assisted by a strong driving field. A number of multiparty entangled states between the atoms and the squeezed vacuum states, among the atoms, and among the squeezed vacuum states, can be prepared by virtue of a specific choice of the cavity detuning and the detuning of applied coherent field under the dissipation condition. The corresponding analytical expressions of the influence can be given. Moreover, we can also give a series of macroscopic entangled states between the usual coherent states and the squeezed vacuum states using the combination of the dissipative one-photon interaction Hamiltonian with the dissipative two-photon interaction Hamiltonian. We also discuss the experimental feasibility. Our scheme can be realized in the current techniques on the cavity QED.

Keywords Multiparty entanglement · Cavity QED · Two-photon interaction

1 Introduction

There are generally two kind of fundamental quantum nonclassical states: *the coherent state* which is made by applying the displacement operator on the vacuum state and *the squeezed vacuum state* which is obtained from the squeezed operator functioned on the vacuum state. Indeed, Schrödinger discussed the nonexistence of quantum superpositions at the classical level in his famous "cat paradox" [1–3], and then stated that if the quantum superposition principles of the quantum dynamics are valid up to the macroscopic level, then the existence of quantum interference at the microscopic level implies that the same phenomenon should occur between distinguishing macroscopic states. Therefore, whether there exists macroscopic quantum coherence becomes the core of debating quantum mechanics. In fact, for

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macroscopic superpositions, quantum coherence decays much faster than the usual physical observables of the system, its decay time being given by the energy dissipation time divided by a dimensionless number measuring the "separation" between the two parts. Naturally, the decoherence process is also at the focus of the quantum measurement. Two types of evolution in quantum mechanics are introduced by Von. Neumann in his collapse postulate [4]: *the deterministic and unitary evolution* associated with the Schrödinger equation, which establishes the correlation between states of microscopic system being measured and distinguishing classical states of the macroscopic measurement apparatus; and *the probabilistic and irreversible process* associated with measurement, which transforms the correlated state into a statistical mixture.

In general, a fundamental issue in quantum measurement theory is the transition between the microscopic and macroscopic worlds, which leads to the extensive studies of mesoscopic quantum states [5–12]. The coherent states have extensively been investigated [5–12], and then the corresponding superpositions and entangled states have been prepared in cavity QED [5–10] and in trapped ions system [11, 12]. While only a little attention has been paid on the study of the squeezed vacuum state, which is another important mesoscopic quantum state. For example, a scheme [13] for squeezed vacuum measurements without homodyning has been proposed by Wenger et al., following the theoretical proposal presented by Fiurášck and Cerf [14]. The scheme can be utilized to measure the squeezing and purity of single mode squeezed vacuum state, providing a powerful tool for the study on the squeezed vacuum state. Recently, Chen et al. have proposed a novel scheme for the generation of superposition and entanglement of mesoscopic squeezed vacuum states in cavity QED by considering the two-photon interaction of N two-level atoms in a cavity with high quality factor, assisted by a strong driving field [15]. In this scheme, the macroscopic squeezed vacuum "Schrödinger cats" and a number of multiparty mesoscopic entangled states can be prepared by virtue of the detuned cavity and the applied coherent field. However, the influence from dissipation on the preparation of them is not considered.

In this paper, we propose a scheme for generation of the macroscopic superpositions and the entanglement between the mesoscopic squeezed vacuum states by considering the two-photon interaction of N two-level atoms in a dissipative cavity with high quality factor assisted by a strong driving field, obtain a number of multiparty entangled states between the atoms and the squeezed vacuum states, among the atoms, and among the squeezed vacuum states, by virtue of a specific choice of the cavity detuning and the detuning of applied coherent field under the dissipation condition, and give the corresponding analytical expressions of the influence of the cavity mode decay on the fidelity F. Moreover, we can also give a series of macroscopic entangled states between the generalized coherent states and the generalized squeezed vacuum states using the combination of the dissipative one-photon interaction Hamiltonian with the dissipative two-photon interaction Hamiltonian. Our scheme can be realized in the current techniques on the cavity QED.

The paper is organized as follows. In Sect. 2, the dissipative Jaynes-Cummings model based on two-photon interaction is investigated and the corresponding effective Hamiltonian is given. In Sect. 3, the entanglement between two mesoscopic squeezed vacuum states under dissipation is discussed. The "Schrödinger cat" state consisting of the generalized squeezed vacuum states is described in Sect. 4. In Sect. 5, entanglement between the generalized coherent states and generalized squeezed vacuum states under dissipation is studied. Finally, some discussion and conclusion are given in Sect. 6.

2 Dissipative Jaynes-Cummings Model Based on Two-Photon Interaction

The Jaynes-Cummings model [16] based on the one-photon interaction can describe the interaction of a two-level atom with a single mode of the electromagnetic field, which is the simplest and most fundamental quantum model. Here, we consider a spatially narrow bunch of N identical two-level atoms in a dissipative two-photon interaction with M modes in a cavity of high quality factor, driven additionally by an external classical field. The Hamiltonian under the cavity mode decay can be expressed as (assuming $\hbar = 1$)

$$H_s = \omega_0 \sum_{j=1}^{N} S_{z,j} + \sum_{i=1}^{M} \omega_{ci} a_i^+ a_i + \sum_{j=1}^{N}\sum_{i=1}^{M} g_{ij}(a_i^{+2} S_j^- + a_i^2 S_j^+) + \Omega \sum_{j=1}^{N} (e^{-i\omega_L t} S_j^+ + e^{i\omega_L t} S_j^-) - i\frac{\kappa}{2}\sum_{i=1}^{M} a_i^+ a_i, \tag{1}$$

where ω_0, ω_{ci}, and ω_L are the frequencies of the resonant transition between $|e\rangle$ and $|g\rangle$, the cavity modes, and the classical laser field, respectively. κ denotes the decay rate of the cavity modes. $S_{z,j} = \frac{1}{2}(|e\rangle\langle e| - |g\rangle\langle g|)$, $S_j^+ = |e\rangle\langle g|$, and $S_j^- = |g\rangle\langle e|$, $a_i^+$and a_i are the creation and annihilation operators for the cavity mode, respectively, and g and Ω are the coupling constants of each atom to the cavity modes and to the driving field, respectively. In the rotating frame with respect to the driving field frequency ω_L, the Hamiltonian is given by

$$H_r = \sum_{j=1}^{N} \Delta S_{z,j} + \sum_{i=1}^{M} \delta_i a_i^+ a_i + \Omega \sum_{j=1}^{N} (S_j^+ + S_j^-) + \sum_{j=1}^{N}\sum_{i=1}^{M} g_{ij}(a_i^{+2} S_j^- + a_i^2 S_j^+) - i\frac{\kappa}{2}\sum_{i=1}^{M} a_i^+ a_i, \tag{2}$$

where $\Delta = \omega_0 - \omega_L$ and $\delta_i = \omega_{ci} - \omega_L/2$. Assume that $\Delta = 0$ is satisfied, in the interaction picture we have

$$\begin{aligned} H_i &= e^{iH_{r0}t} H_{ri} e^{-iH_{r0}t} \\ &= \frac{1}{2}\sum_{j=1}^{N}\sum_{i=1}^{M} g_{ij}[|+\rangle_{jj}\langle +| - |-\rangle_{jj}\langle -| + e^{i2\Omega t}|+\rangle_{jj}\langle -| \\ &\quad - e^{-i2\Omega t}|-\rangle_{jj}\langle +|]a_i^2 e^{-i2\delta_i t} e^{-\kappa t} \\ &\quad + \frac{1}{2}\sum_{j=1}^{N}\sum_{i=1}^{M} g_{ij}[|+\rangle_{jj}\langle +| - |-\rangle_{jj}\langle -| + e^{-i2\Omega t}|-\rangle_{jj}\langle +| \\ &\quad + -e^{i2\Omega t}|+\rangle_{jj}\langle -|]a_i^{+2} e^{i2\delta_i t} e^{\kappa t}, \end{aligned} \tag{3}$$

where $H_r = H_{r0} + H_{ri}$,

$$H_{r0} = \sum_{i=1}^{M} \delta_i a_i^+ a_i + \Omega \sum_{j=1}^{N} (S_j^+ + S_j^-) - i\frac{\kappa}{2}\sum_{i=1}^{M} a_i^+ a_i, \tag{4}$$

and

$$H_{ri} = \sum_{j=1}^{N}\sum_{i=1}^{M} g_{ij}(a_i^{+2}S_j^- + a_i^2 S_j^+), \tag{5}$$

and where $|\pm\rangle_j = (|g\rangle_j \pm |e\rangle_j)/\sqrt{2}$ and $S_{jx}|\pm\rangle_j = (S_j^+ + S_j^-)|\pm\rangle_j = \pm|\pm\rangle_j$. In the strong driving regime $\Omega \gg \delta_i, g_{ij}, \kappa$, a rotating-wave approximation can be utilized and then the effective Hamiltonian is obtained as follows

$$\begin{aligned} H_{ieff} &= \frac{1}{2}\sum_{j=1}^{N}\sum_{i=1}^{M} g_{ij}(|+\rangle_{jj}\langle+| - |-\rangle_{jj}\langle-|)(a_i^2 e^{-i2\delta_i t}e^{-\kappa t} + a_i^{+2}e^{i2\delta_i t}e^{\kappa t}) \\ &= \frac{1}{2}\sum_{j=1}^{N}\sum_{i=1}^{M} S_{jx} g_{ij}(a_i^2 e^{-i2\delta_i t}e^{-\kappa t} + a_i^{+2}e^{i2\delta_i t}e^{\kappa t}). \end{aligned} \tag{6}$$

3 Entanglement between the Two Mesoscopic Squeezed Vacuum States under Dissipation

In this section, we will use (6) to generate the entangled state between the two mesoscopic squeezed vacuum states under the considering of the cavity mode decay. We consider the case that $N = 1, M = 2$, being two quasiresonant modes in the cavity, and the atom-field is initially in $|g\rangle|0\rangle_1|0\rangle_2$. After the small interaction t, the total state evolves into

$$\begin{aligned} |\psi\rangle &= e^{-iH_{ieff}t}|g\rangle|0\rangle_1|0\rangle_2 \\ &= e^{-iH_{ieff}t}\frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)|0\rangle_1|0\rangle_2 \\ &= \frac{1}{\sqrt{2}}[|+\rangle S_g[\xi_1(t), \xi_1(-t)]S_g[\xi_2(t), \xi_2(-t)] \\ &\quad + |-\rangle S_g[-\xi_1(t), -\xi_1(-t)]S_g[-\xi_2(t), -\xi_2(-t)]]|0\rangle_1|0\rangle_2 \\ &= \frac{1}{\sqrt{2}}(|+\rangle|\xi_1\rangle_g|\xi_2\rangle_g + |-\rangle|-\xi_1\rangle_g|-\xi_2\rangle_g), \end{aligned} \tag{7}$$

where the generalized squeezed operator is introduced and defined by $S_g[\xi_l(t), \xi_l(-t)] = e^{\frac{1}{2}[\xi_l(t)a_l^2 + \xi_l(-t)a_l^{+2}]}$, different from the standard squeezed operator $S_s[\xi_l'(t)] = e^{\frac{1}{2}[\xi_l'(t)a_l^2 - \xi_l'^*(t)a_l^{+2}]}$, $\xi_l(t) = -\int_0^t i g_l e^{-i2\delta_i t'}e^{-\kappa t'}dt' = g_l[2\delta_i + i\kappa][e^{-\kappa t}e^{-i2\delta_i t} - 1]/[4\delta_i^2 + \kappa^2]$, $(l = 1, 2)$ and $|\xi_l\rangle_g$ is the generalized squeezed vacuum states of the cavity modes, which is different from $|\xi_l'\rangle_s = S_s[\xi_l'(t)]|0\rangle$. Equation (7) describes a generalized three-party entangled state between one microscopic and two mesoscopic systems. If the cavity mode decay rate $\kappa = 0$, (7) will return to the ideal case $|\psi\rangle_{ideal}$, which is expressed as

$$|\psi\rangle_{ideal} = \frac{1}{\sqrt{2}}(|+\rangle|\xi_1'\rangle_s|\xi_2'\rangle_s + |-\rangle|-\xi_1'\rangle_s|-\xi_2'\rangle_s). \tag{8}$$

Therefore, we can give the fidelity of generation of the three-party entangled state in (8):

$$
\begin{aligned}
F_1 &= |_{ideal}\langle\psi||\psi\rangle|^2 \\
&= \frac{1}{4}\{|_s\langle\xi_1^{'}||\xi_1\rangle_{gs}\langle\xi_2^{'}||\xi_2\rangle_g|^2 + |_s\langle-\xi_1^{'}||-\xi_1\rangle_{gs}\langle-\xi_2^{'}||-\xi_2\rangle_g|^2 \\
&\quad + 2_s\langle\xi_1^{'}||\xi_1\rangle_{gs}\langle\xi_2^{'}||\xi_2\rangle_{gs}\langle-\xi_1^{'}||-\xi_1\rangle_{gs}\langle-\xi_2^{'}||-\xi_2\rangle_g\}, \\
&= |_s\langle\xi_1^{'}||\xi_1\rangle_g|^2|_s\langle\xi_2^{'}||\xi_2\rangle_g|^2 \\
&= \cosh|\xi_1^{'}|\cosh|\xi_2^{'}|\cosh|\xi_1|\cosh|\xi_2| \\
&\quad \times \left|\sum_{m,n,k,l}^{\infty}\frac{e^{i(m+n+k+l)(\theta_1+\theta_2+\theta_1^{'}+\theta_2^{'})}(\tanh|\xi_1^{'}|\tanh|\xi_2^{'}|\tanh|\xi_1|\tanh|\xi_2|)^{(m+n+k+l)}}{2^{2(m+n+k+l)}(2m)!(2n)!(2k)!(2l)!}\right|,
\end{aligned} \tag{9}
$$

where

$$
\begin{aligned}
&\xi_i^{'}(t) = g_i(e^{-i2\delta_i t}-1)/(4\delta_i^2+\kappa^2), (i=1,2), \\
&\xi_i(t) = g_i(2\delta_i+i\kappa)(e^{-\kappa t}e^{-i2\delta_i t}-1)/(4\delta_i^2+\kappa^2) \quad (i=1,2), \\
&|\xi_i^{'}(t)| = \left|\frac{g_i\sin(\delta_i t)}{\delta_i}\right|, \\
&|\xi_i(t)| = \frac{g_i}{4\delta_i^2+\kappa^2}\{[2\delta_i e^{-\kappa t}\sin 2\delta_i t - \kappa(e^{-\kappa t}\cos 2\delta_i t - 1)]^2 \\
&\qquad + [2\delta_i(e^{-\kappa t}\cos 2\delta_i t - 1) - \kappa e^{-\kappa t}\sin 2\delta_i t]^2\}^{\frac{1}{2}}, \\
&\theta_i^{'}(t) = -\arctan(\tan^{-1}\delta_i t), \\
&\theta_i(t) = \arctan\frac{2\delta_i(e^{-\kappa t}\cos 2\delta_i t - 1) - \kappa e^{-\kappa t}\sin 2\delta_i t}{2\delta_i e^{-\kappa t}\sin 2\delta_i t - \kappa(e^{-\kappa t}\cos 2\delta_i t - 1)}.
\end{aligned} \tag{10}
$$

If $\xi_1^{'} = \xi_2^{'}$ and $\xi_1 = \xi_2$, through complex derivation, we can obtain

$$
F_1 = (\cosh|\xi_1^{'}|\cosh|\xi_1|)^2 \times \left|\sum_{m,n}^{\infty}\frac{e^{i(m+n)(\theta_1+\theta_1^{'})}(\tanh|\xi_1^{'}|\tanh|\xi_1|)^{(m+n)}}{2^{2(m+n+k+l)}(2m)!(2n)!}\right|^2. \tag{11}
$$

According to Ref. [15], (7) can also be written as

$$
\begin{aligned}
|\psi\rangle = \frac{1}{2}[|g\rangle(|\xi_1\rangle_g|\xi_2\rangle_g + |-\xi_1\rangle_g|-\xi_2\rangle_g) \\
+ |e\rangle(|\xi_1\rangle_g|\xi_2\rangle_g - |-\xi_1\rangle_g|-\xi_2\rangle_g).
\end{aligned} \tag{12}
$$

Then the atom is detected in the basis $\{|g\rangle, |e\rangle\}$, we can obtain the macroscopic entangled squeezed vacuum states

$$
|\psi\rangle_{\pm} = \frac{1}{\sqrt{2}}[|\xi_1\rangle_g|\xi_2\rangle_g \pm |-\xi_1\rangle_g|-\xi_2\rangle_g], \tag{13}
$$

respectively. If the atom pass continuously through the n same cavities as above before the measurement, we can acquire more macroscopic entangled squeezed states after the detection of the atom

$$
|\psi\rangle_{\pm} = \frac{1}{\sqrt{2^n}}\left[\prod_{j=1}^{2n}|\xi_j\rangle_g \pm (-1)^n\prod_{j=1}^{2n}|-\xi_j\rangle_g\right]. \tag{14}
$$

It is noted that this is a more macroscopic entangled squeezed state involving in $2n$ mesoscopic systems, called as generalized macroscopic *GHZ* state in which the cavity mode decay has been considered. The corresponding fidelity is $(F_1)^n$ when $\xi_j' = \xi_1'$ and $\xi_j = \xi_1, j = 1, \ldots, 2n$.

4 "Schrödinger Cat" State Consisting of the Generalized Squeezed Vacuum States

We turn to study the case of an atom interacting with a single mode, in order to generate the "Schrödinger cat" state consisting of the generalized squeezed vacuum states. Given that the atom-field state is initially $|g\rangle_1|0\rangle_1$, after the interaction t, the system evolves to

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|+\rangle_1|\xi_1\rangle_g + |-\rangle_1|-\xi_1\rangle_g), \tag{15}$$

where $\xi_1(t) = ig[2\delta_1 + i\kappa][e^{-\kappa t}e^{-i2\delta_1 t} - 1]/[4\delta_1^2 + \kappa^2]$, $\xi_1(-t) = ig[2\delta_1 + i\kappa][e^{\kappa t}e^{i2\delta_1 t} - 1]/[4\delta_1^2 + \kappa^2]$ and the state is the microscopic-mesoscopic entangled state between the atomic internal states and the generalized squeezed states, usually called the "Schrödinger cat" state, different from those made from the displaced coherent states [4, 5, 15]. If $\kappa = 0$, we have

$$|\psi\rangle_{ideal} = \frac{1}{\sqrt{2}}(|+\rangle_1|\xi_1'\rangle_s + |-\rangle_1|-\xi_1'\rangle_s). \tag{16}$$

With the same method as above, the fidelity can be given

$$\begin{aligned} F_2 &= |_s\langle\xi_1'||\xi_1\rangle_g|^2 \\ &= (\cosh|\xi_1'|\cosh|\xi_1|) \times \left|\sum_{m,n}^{\infty} \frac{e^{i(m+n)(\theta_1+\theta_1')}(\tanh|\xi_1'|\tanh|\xi_1|)^{(m+n)}}{2^{2(m+n)}(2m)!(2n)!}\right|. \end{aligned} \tag{17}$$

We obtain also from (15)

$$|\psi\rangle = \frac{1}{2}[|g\rangle_1(|\xi_1\rangle_g + |-\xi_1\rangle_g) + |e\rangle_1(|\xi_1\rangle_g - |-\xi_1\rangle_g)], \tag{18}$$

showing that the detection of the atomic state will create the *even* or *odd* squeezed vacuum states in the dissipative cavity field, relying on whether $|g\rangle_1$ or $|e\rangle_1$ was found, respectively. The states can also be expressed as follows

$$|\psi\rangle_{1eoc} = \frac{1}{\sqrt{2}}(|\xi_1\rangle_g \pm |-\xi_1\rangle_g). \tag{19}$$

If we consider that the two-atom-field state is initially in $|g\rangle_1|g\rangle_2|0\rangle_1$, in the interaction picture, after a time t, the evolved state will be

$$\begin{aligned} |\psi\rangle_{2i} &= \frac{1}{2}(|+\rangle_1|+\rangle_2|2\xi_1\rangle_g + (|+\rangle_1|-\rangle_2 + |-\rangle_1|+\rangle_2)|0\rangle_1 + |-\rangle_1|-\rangle_2|-2\xi_1\rangle_g) \\ &= \frac{1}{2}\{|g\rangle_1|g\rangle_2[|2\xi_1\rangle_g + 2|0\rangle_1 + |-2\xi_1\rangle_g] + |g\rangle_1|e\rangle_2[|2\xi_1\rangle_g - |-2\xi_1\rangle_g] \\ &\quad + |e\rangle_1|g\rangle_2[|2\xi_1\rangle_g - |-2\xi_1\rangle_g] + |e\rangle_1|e\rangle_2[|2\xi_1\rangle_g - 2|0\rangle_1 + |-2\xi_1\rangle_g]. \end{aligned} \tag{20}$$

Detecting the two atom in the states $|g\rangle_1|g\rangle_2$, $|g\rangle_1|e\rangle_2(|e\rangle_1|g\rangle_2)$, and $|e\rangle_1|e\rangle_2$ will generate the field states, respectively

$$
\begin{aligned}
|\psi\rangle_{2s1} &= \frac{1}{\sqrt{2}}(|2\xi_1\rangle_g + 2|0\rangle_1 + |-2\xi_1\rangle_g),\\
|\psi\rangle_{2s3} &= \frac{1}{\sqrt{2}}(|2\xi_1\rangle_g - |-2\xi_1\rangle_g),\\
|\psi\rangle_{2s3} &= \frac{1}{\sqrt{2}}(|2\xi_1\rangle_g - 2|0\rangle_1 + |-2\xi_1\rangle_g).
\end{aligned}
\tag{21}
$$

Naturally, supposing that the $(2n+1)$-atom-field is initially in $\prod_{i}^{2n+1}|g\rangle_i|0\rangle_1$, through the same derivation as above, we can obtain the evolved state

$$
\begin{aligned}
|\psi\rangle_{2n+1i} = \frac{1}{\sqrt{2^n}}\Bigg[&\prod_{i=1,}^{2n+1}|+\rangle_i|(2n+1)\xi_1\rangle_g + \sum_{k\neq i}^{2n+1}\prod_{i=1,}^{2n+1}|+\rangle_i|-\rangle_k|(2n-1)\xi_1\rangle_g\\
&+\cdots+\sum_{i\neq k}^{2n+1}\prod_{k=1,}^{2n+1}|+\rangle_i|-\rangle_k|-(2n-1)\xi_1\rangle_g\\
&+\prod_{k=1,}^{2n+1}|-\rangle_k|-(2n+1)\xi_1\rangle_g\Bigg].
\end{aligned}
\tag{22}
$$

If the measurement of the atoms is in the $\prod_{i=1}^{2n+1}|g\rangle_i$, we have

$$
|\psi\rangle_{2n+1s} = \sum_{j=-(n+1)}^{n} C_{2n+1}^{n-j}|(2j+1)\xi_1\rangle_g, \tag{23}
$$

which is a superposition state compositing of $2(n+1)$-mesoscopic fields without an Fock state $|0\rangle$. In fact, if the initial state $\prod_{i}^{2n}|g\rangle_i|0\rangle$ is considered, we can acquire

$$
|\psi\rangle_{2ns} = \sum_{j=-n}^{n} C_{2n}^{n-j}|2j\xi_1\rangle_g, \tag{24}
$$

which is $(2n+1)$-mesoscopic field superposition state involving the Fock state $|0\rangle$. We usually call the states in (23) and (24) as *the generalized bigger macroscopic "Schrödinger cat" states* [1–3].

5 Entanglement between the Generalized Coherent States and Generalized Squeezed Vacuum States under Dissipation

We now turn to prepare the entangled states between the generalized coherent states and the generalized squeezed vacuum states in the case of cavity mode dissipation. Assume that a spatially narrow bunch of N identical two-level atoms of one-photon-interaction with M modes in a cavity of high quality factor and with cavity mode decay rate κ, is driven additionally by an external classical field. The Hamiltonian can be expressed as (assuming $\hbar = 1$)

$$H'_{so} = \omega_0 \sum_{j=1}^{N} S_{z,j} + \sum_{i=1}^{M} \omega_{ci} a_i^+ a_i + \sum_{j=1}^{N}\sum_{i=1}^{M} g_{ij}(a_i^+ S_j^- + a_i S_j^+)$$
$$+ \Omega \sum_{j=1}^{N} (e^{-i\omega'_L t} S_j^+ + e^{i\omega'_L t} S_j^-) - i\frac{\kappa}{2}\sum_{i=1}^{M} a_i^+ a_i, \tag{25}$$

where ω_0, ω_{ci}, and ω'_L are the frequencies of the resonant transition between $|e\rangle$ and $|g\rangle$, the cavity mode, and the classical laser field, respectively. $S_{z,j} = \frac{1}{2}(|e\rangle\langle e| - |g\rangle\langle g|)$, $S_j^+ = |e\rangle\langle g|$, and $S_j^- = |g\rangle\langle e|$, a_i^+ and a_i are the creation and annihilation operators for the cavity mode, respectively, and g and Ω are the coupling constants of the interaction of easc atom with the cavity modes and with the driving field, respectively. In the rotating frame with respect to the driving field frequency ω'_L, the Hamiltonian is given by

$$H'_r = \sum_{j=1}^{N} \Delta' S_{z,j} + \sum_{i=1}^{M} \delta_i a_i^+ a_i + \Omega \sum_{j=1}^{N} (S_j^+ + S_j^-)$$
$$+ \sum_{j=1}^{N}\sum_{i=1}^{M} g_{ij}(a_i^+ S_j^- + a_i S_j^+) - i\frac{\kappa}{2}\sum_{i=1}^{M} a_i^+ a_i, \tag{26}$$

where $\Delta' = \omega_0 - \omega'_L$ and $\delta'_i = \omega_{ci} - \omega'_L$. Assume that $\Delta' = 0$ is satisfied, in interaction picture we have

$$\begin{aligned} H'_i &= e^{iH_{r0}t} H'_{ri} e^{-iH_{r0}t} \\ &= \frac{1}{2}\sum_{j=1}^{N}\sum_{i=1}^{M} g_{ij}[|+\rangle_{jj}\langle+| - |-\rangle_{jj}\langle-| + e^{i2\Omega t}|+\rangle_{jj}\langle-| \\ &\quad - e^{-i2\Omega t}|-\rangle_{jj}\langle+|]a_i e^{-i\delta t} e^{-\kappa t/2} \\ &\quad + \frac{1}{2}\sum_{j=1}^{N}\sum_{i=1}^{M} g_{ij}[|+\rangle_{jj}\langle+| - |-\rangle_{jj}\langle-| \\ &\quad + e^{-i2\Omega t}|-\rangle_{jj}\langle+| - e^{i2\Omega t}|+\rangle_{jj}\langle-|]a_i^+ e^{i\delta t} e^{\kappa t/2}, \end{aligned} \tag{27}$$

where

$$H'_{r0} = \sum_{i=1}^{M} \delta_i a_i^+ a_i + \Omega \sum_{j=1}^{N} (S_j^+ + S_j^-) - i\frac{\kappa}{2}\sum_{i=1}^{M} a_i^+ a_i, \tag{28}$$

and

$$H'_{ri} = \sum_{j=1}^{N}\sum_{i=1}^{M} g_{ij}(a_i^+ S_j^- + a_i S_j^+), \tag{29}$$

and where $|\pm\rangle_j = (|g\rangle_j \pm |e\rangle_j)/\sqrt{2}$ and $S_{jx}|\pm\rangle_j = (S_j^+ + S_j^-)|\pm\rangle_j = \pm|\pm\rangle_j$. In the strong driving regime $\Omega \gg \delta_i, g_{ij}$, a rotating-wave approximation can be realized and then the effective Hamiltonian is obtained as follows

$$\begin{aligned} H'_{ieff} &= \frac{1}{2}\sum_{j=1}^{N}\sum_{i=1}^{M} g_{ij}(|+\rangle_{jj}\langle+| - |-\rangle_{jj}\langle-|)(a_i e^{-i\delta_i t}e^{-\kappa t/2} + a_i^+ e^{i\delta_i t}e^{\kappa t/2}) \\ &= \frac{1}{2}\sum_{j=1}^{N}\sum_{i=1}^{M} S_{jx} g_{ij}(a_i e^{-i\delta_i t}e^{-\kappa t/2} + a_i^+ e^{i\delta_i t}e^{\kappa t/2}). \end{aligned} \tag{30}$$

Apparently, we can use (31) to generate the *even* or *odd* coherent states of dissipation, the "Schrödinger cat" states of the generalized coherent states, and the entangled states between the generalized coherent states [8], similar to those in (14–17), (19), (20), respectively. Supposing that one atom passes sequentially through cavity 1 for interaction time t_1 and cavity 2 for interaction time t_2, the interactions between the atom and the cavity 1 and the cavity 2 are controlled by (31) and (6), respectively. If the system is initially in $|g\rangle|0\rangle_1|0\rangle_2$, after the interaction time $t_1 + t_2$, we can obtain in the interaction picture

$$|\psi\rangle' = \frac{1}{\sqrt{2}}(|+\rangle_1|\alpha_1\rangle_g|\xi_2\rangle_g + |-\rangle|-\alpha_1\rangle_g|-\xi_2\rangle_g), \tag{31}$$

where $\alpha_1(t_1) = (2g_{11}\delta_1 - ig_{11}\kappa)(e^{i\delta_1 t_1}e^{-\kappa t_1/2} - 1)/(4\delta_1^2 + \kappa^2)$, $\alpha_1(-t_1) = (2g_{11}\delta_1 - ig_{11}\kappa)$ $(e^{-i\delta_1 t_1}e^{\kappa t_1/2} - 1)/(4\delta_1^2 + \kappa^2)$, $|\alpha_1\rangle_g = D_g[\alpha_1(t_1), \alpha_1(-t_1)]|0\rangle_1 = e^{[\alpha_1(t_1)a_1^+ + \alpha_1(-t_1)a_1]}|0\rangle_1$, in which $D_g[\alpha_1(t_1), \alpha_1(-t_1)]$ is the like-displacement operator first introduced by us in Ref. [9, 10], and $\xi_2(t_2)$ is showed as above. This is a generalized tri-party *GHZ* state, which includes a microscopic state (the atomic electronic state) and two different mesoscopic states (i.e., the generalized coherent state and the generalized squeezed vacuum state). Its fidelity can be obtained

$$\begin{aligned} F_3 &= |_s\langle\alpha_1'||\alpha_1\rangle_g|^2|_s\langle\xi_2'||\xi_2\rangle_g|^2 \\ &= \exp\left\{-\frac{g_1^2}{(4\delta_1^2+\kappa^2)^2}\left\{\left[2\delta_1(e^{-\kappa t/2}\cos\delta_1 t - 1)\right.\right.\right. \\ &\quad \left. + \kappa e^{-\kappa t/2}\sin\delta_1 t - \frac{4\delta_1^2+\kappa^2}{2\delta_1}(\cos\delta_1 t - 1)\right]^2 \\ &\quad \left.\left. + \left[2\delta_1 e^{-\kappa t/2}\sin\delta_1 t - \kappa(e^{-\kappa t/2}\cos\delta_1 t - 1) - \frac{4\delta_1^2+\kappa^2}{4\delta_1}\sin\delta_1 t\right]^2\right\}\right\} \\ &\quad \times(\cosh|\xi_2'|\cosh|\xi_2|) \times \left|\sum_{m,n}^{\infty}\frac{e^{i(m+n)(\theta_1+\theta_1')}(\tanh|\xi_2'|\tanh|\xi_2|)^{(m+n)}}{2^{2(m+n)}(2m)!(2n)!}\right|, \end{aligned} \tag{32}$$

where

$$\begin{aligned}
&\alpha_1'(t) = g_1(e^{i\delta_1 t} - 1)/2\delta_1,\\
&\alpha_1(t) = g_1(2\delta_1 - i\kappa)(e^{-\kappa t/2}e^{i\delta_1 t} - 1)/(4\delta_1^2 + \kappa^2),\\
&|\xi_2'(t)| = \left|\frac{g_2\sin(\delta_2 t)}{\delta_2}\right|,\\
&|\xi_2(t)| = \frac{g_2}{4\delta_2^2+\kappa^2}\{[2\delta_2 e^{-\kappa t}\sin 2\delta_2 t - \kappa(e^{-\kappa t}\cos 2\delta_2 t - 1)]^2\\
&\qquad + [2\delta_2(e^{-\kappa t}\cos 2\delta_2 t - 1) - \kappa e^{-\kappa t}\sin 2\delta_2 t]^2\}^{\frac{1}{2}},\\
&\theta_1'(t) = -\arctan\left(\tan^{-1}\frac{\delta_1 t}{2}\right),\\
&\theta_1(t) = \arctan\frac{2\delta_1 e^{-\kappa t/2}\sin\delta_1 t - \kappa(e^{-\kappa t/2}\cos\delta_1 t - 1)}{2\delta_1(e^{-\kappa t/2}\cos\delta_1 t - 1) + \kappa e^{-\kappa t/2}\sin\delta_1 t}.
\end{aligned} \tag{33}$$

We can also have from (31)

$$\begin{aligned}
|\psi\rangle' = \frac{1}{2}[&|g\rangle(|\alpha_1\rangle_g|\xi_2\rangle_g + |-\alpha_1\rangle_g|-\xi_2\rangle_g)\\
&+ |e\rangle(|\alpha_1\rangle_g|\xi_2\rangle_g - |-\alpha_1\rangle_g|-\xi_2\rangle_g)].
\end{aligned} \tag{34}$$

When the measurement of the atom is performed in $\{|g\rangle, |e\rangle\}$, the generalized macroscopic entangled state between the coherent state and the squeezed state can be produced

$$|\psi\rangle_{mBell} = \frac{1}{\sqrt{2}}(|\alpha_1\rangle_g|\xi_2\rangle_g \pm |-\alpha_1\rangle_g|-\xi_2\rangle_g). \tag{35}$$

This state is also called as the *generalized macroscopic Bell states* composed of two types different mesoscopic quantum states, which may be of extensive applications in the quantum information processing. We can generalize the method above to acquire *the generalized macroscopic GHZ states*

$$|\psi\rangle_{mGHZ} = \frac{1}{\sqrt{2}}\left(\prod_{j=1}^{n}|\alpha_{2j-1}\rangle_g|\xi_{2j}\rangle_g \pm \prod_{j=1}^{n}|-\alpha_{2j-1}\rangle_g|-\xi_{2j}\rangle_g\right), \tag{36}$$

where the interactions of the atom with cavities $2j-1$ and $2j$ are determined by the Hamiltonians in (31) and (6), respectively. Indeed, the macroscopic entangled states of arbitrary coherent states and arbitrary squeezed vacuum states can be generated using this method, which can be utilized to test the quantum nonlocality of quantum mechanics. Moreover, we can implement the entangled states between the bigger Schrödinger cat states made from the generalized coherent states and the generalized squeezed vacuum states.

It is worthwhile mentioned that when $\delta = \pm 2\Omega$ and $|\delta| \gg g_i$, in the dressed basis $|\pm\rangle_j$, (3) will turn into

$$H_{TJC}^{+} = \frac{1}{2}\sum_{j=1}^{N}\sum_{i=1}^{M} g_{ij}[|+\rangle_{jj}\langle -|a_i^2 e^{-i2\delta_i t}e^{-\kappa t} + |-\rangle_{jj}\langle +|a_i^{+2}e^{i2\delta_i t}e^{\kappa t}]), \tag{37}$$

and

$$H^{-}_{TAJC} = \frac{1}{2}\sum_{j=1}^{N}\sum_{i=1}^{M} g_{ij}[|+\rangle_{jj}\langle -|a_i^{+2}e^{i2\delta_i t}e^{\kappa t} + |-\rangle_{jj}\langle +|a_i^2 e^{-i2\delta_i t}e^{-\kappa t}], \tag{38}$$

where H^{+}_{TJC} and H^{-}_{TAJC} represent effective realization of a dissipative two-photon Jaynes-Cummings and a dissipative two-photon anti-Jaynes-Cummings interactions with multi-modes in the cavity, the applications of which will be discussed otherwhere.

6 Discussion of the Experiment Feasibility and Conclusion

Our proposal is experimentally feasible with current available cavity QED techniques. Both microwave and optical regimes may be utilized for implementation of our scheme in the cases of both open and closed cavities [19, 20]. For example, in the microwave regime, we assume that $\delta/2\pi = 1$ MHz, $\Omega/2\pi = 1$ GHz, and $g_{ij}/2\pi = 0.05$ MHz [19, 20]. The lifetime of the circular Rydberg atom state with principle quantum 51 is about 30 ms, much longer than the cavity decay time T_{cav}(0.85 ms) in the case of multiphoton inside [21]. If we assume that the cavity size is $L = 27.5$ mm and the atomic velocity is 500 m/s [22, 23], the interaction time of the atoms with the cavity mode is $T_i = 5.5 \times 10^{-2}$ ms, which is much shorter than T_{cav}, and then $T_{cav}/T_i = 15.45$, meaning that if the detection time T_d is equal to the interaction time T_i, we can sequentially manipulate about *five–six* atoms to realize our scheme in a single cavity. Furthermore, if the distance between neighbor cavities is half of the cavity size, we can implement the multi-cavity part of our scheme with about *six–seven* cavities involved by detection on a single atom.

In conclusion, we have been proposed a feasible scheme for realizing the generalized superpositions and entanglement of the mesoscopic squeezed vacuum states. In our scheme, we have utilized the interactions between many atoms and two cavity modes in a cavity or many cavity modes in many cavities of dissipations, which are based on the dissipative two-photon Jayness-Cumming model assisted by strongly-assisted driving classical field. We can also give a series of macroscopic entangled states between the generalized coherent states and the generalized squeezed vacuum states using the combination of the dissipative one-photon interaction Hamiltonian with the dissipative two-photon interaction Hamiltonian. The analytical expressions of the fidelities for generation of superpositions and entanglement of generalized squeezed vacuum states and generalized coherent states of the cavity fields of dissipation have been shown. Alternatively, we have introduced the concepts of the generalized squeezed operator, the generalized displaced operator, generalized squeezed state and generalized coherent state, which is useful for investigating the situation of dissipation. All obtained results are suitable for realization in the microscopic and optical regimes in cavity QED experiments, with atoms flying through the cavities or conveniently trapped inside them [8]. We argue that our schemes can be realized by current techniques in cavity QED [16–20].

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References

1. Schrödinger, E.: Naturwissenschaften **23**, 807 (1935)

2. Schrödinger, E.: Naturwissenschaften **23**, 823 (1935)
3. Schrödinger, E.: Naturwissenschaften **23**, 844 (1935)
4. Von Neumann, J.: Die Mathmatisch Grundagen der Quantenmechanik. Springer, Berlin (1932)
5. Davidovich, L., et al.: Phys. Rev. A **53**, 1295 (1996)
6. Brune, M., et al.: Phys. Rev. Lett. **77**, 4887 (1996)
7. Lutterbach, L.G., Davidovich, L.: Phys. Rev. A **61**, 023813 (2000)
8. Solano, E., Agarwal, G.S., Walther, H.: Phys. Rev. Lett. **90**, 027903 (2003)
9. Myatt, C.J., et al.: Nature **403**, 269 (2000)
10. Chen, C.-Y., Feng, M., Zhang, X.-L., Gao, K.-L.: Phys. Rev. A **73**, 032344 (2006)
11. Solano, E., de Matos Filho, R.L., Zagury, N.: Phys. Rev. Lett. **87**, 060402 (2001)
12. Gou, S.-C., Steinbach, J., Knight, P.L.: Phys. Rev. A **55**, 3719 (1997)
13. Wenger, J., et al.: Phys. Rev. A **70**, 053812 (2004)
14. Fiurášek, J., Cerf, N.J.: Phys. Rev. Lett. **93**, 0836001 (2004)
15. Chen, C.-Y., Feng, M., Gao, K.-L.: Phys. Rev. A **73**, 034305 (2006)
16. Jaynes, E.T., Cummings, F.W.: Proc. IEEE **51**, 89 (1963)
17. Osnaghi, S., et al.: Phys. Rev. Lett. **87**, 037902 (2001)
18. Raimond, J.M., Brune, M., Haroche, S.: Rev. Mod. Phys. **73**, 565 (2001)
19. Varcoe, M.T.H., et al.: Nature (London) **403**, 743 (2000)
20. Schmidt, P.O., et al.: Science **309**, 749 (2005)
21. Meunier, T., et al.: Phys. Rev. Lett. **94**, 010401 (2005)
22. Agarwal, G.S., et al.: Phys. Rev. A **48**, 4555 (1993)
23. Pinkse, P.W.H., et al.: Nature (London) **404**, 365 (2000)